

# A Novel Discretization and Numerical Solver for Non-Fourier Diffusion

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## 1 Description

This is the opensource code for SIGGRAPH Asia 2020 submission:

We introduce a generalized constitutive model for diffusion-driven problems with several attractive properties: - It fundamentally explains diffusion from the perspective of the Boltzmann Transport Equation - It can address the issue of infinite propagation speed for diffusion, an assumption made by the widely employed Fick's/Fourier's law. - It can capture some of the most characteristic, yet unprecedented, visual aspects of diffusion-driven physics.

In the algorithm, we propose a novel semi-implicit discretization for this diffusion model on staggered MACgrids and design a geometric Multigrid-preconditioned Conjugate Gradients solver for efficient numerical solution.

## 2 Compilation

The following instruction is tested on Ubuntu 18.04 LTS with g++7.5

### 2.1 Dependencies

#### 2.1.1 Install Nova library and OpenGL viewer

The Nova library can be downloaded from GitHub using the following command:

```
git clone https://github.com/OrionQuest/Nova.git
```

Example projects that use the Nova core library live in a separate repository, and can be downloaded using the following commands:

```
cd Nova/Projects
git clone https://github.com/OrionQuest/Nova_Examples.git
```

Nova depends on several libraries such as GLM,FreeType,GLFW,GLEW,Boost,Assimp, etc. Run the following command to install all the dependencies:

```

sudo apt-get install libboost1.62-dev libboost-program-options1.62-dev
sudo apt-get install libboost-filesystem1.62-dev libboost-regex1.62-dev
sudo apt-get install libglfw3-dev libglew-dev libglm-dev
sudo apt-get install libassimp-dev libfreetype6-dev

```

The build system uses cmake (version 3.0 or higher). We recommend the use of the graphical version ccmake for easy configuration of the environment variables. The best way to install the latest version of cmake and ccmake is to first run the following command:

```

sudo apt-get install cmake
sudo apt-get install cmake-qt-gui
sudo apt-get install libncurses5-dev

```

### 2.1.2 Extract Project

Extract non\_ficks\_diffusion.zip to Nova/Projects. The partial directory should look like this:

```

.
+-- Projects
|   +-- non_ficks_diffusion
|   +-- Nova_Examples
|   +-- CMakeLists.txt
...

```

## 2.2 Build

Run the following commands in order:

```

cd Nova
mkdir build
cd build
ccmake ..

```

Set CMAKE\_BUILD\_TYPE to RELEASE, and turn OFF the following flags: ENABLE\_OPENIMAGEIO, ENABLE\_SOIL\_PLUGIN, ENABLE\_VARIATIONAL\_FLUIDS, USE\_C11\_REGEX, USE\_DOUBLES. Turn all the other flags ON. Press c to configure, and then g to generate the Makefile. Finally, run the following command:

```

make -j 8

```

## 2.3 Configure OpenGL

Run the following commands in order:

```

cd build/
ln -s ../Projects/Nova_Examples/opengl/example/nova.conf .
ln -s ../Projects/Nova_Examples/opengl/example/fonts/ .
ln -s ../Projects/Nova_Examples/opengl/plugins/Grid/shaders/ .

```

Choose the correct plugin for visualization in nova.conf in Nova/build/

### 2.3.1 For hydrogel visualization

libplugin\_Non\_Ficks\_Diffusion has been designed to visualize particles as well as the background grid. The plugins in nova.conf should look like below.

```
...
#Plugin=libplugin_Embedded_Deformables
Plugin=libplugin_Non_Ficks_Diffusion
#Plugin=libplugin_Autonomous_Navigation
...
```

### 2.3.2 For other simulations

libplugin\_Autonomous\_Navigation can be used to show volumetric data. The plugins in nova.conf should look like below.

```
...
#Plugin=libplugin_Embedded_Deformables
#Plugin=libplugin_Non_Ficks_Diffusion
Plugin=libplugin_Autonomous_Navigation
...
```

Here are some of the command options for both plugins.

Command	Description
ALT-P	switch to planar camera
P	play/pause
S	next frame
SHIFT+S	previous frame
R	reset frame
F2	show/hide background grid
Scroll	zoom in/out
Left-mouse	rotate
Right-mouse	translate

## 2.4 Demos

### 2.4.1 Smoke

**Fourier Diffusion** Go to the build directory (Nova/build/), run the following command for Fourier diffusion test case:

```
./bin/smoke_3d -threads 8 -last_frame 500 -test_number 3 -diff_coeff 0.01 \
    -bv 1 -sr 50 -size 256 512 256 -mg_levels 4 -ficks
```

Set plugin to be libplugin\_Autonomous\_Navigation in Nova/build/nova.conf. Use opengl viewer to check out the result, the diffusive smoke:

```
./bin/opengl Implicit_Smoke_2d_F_case_5_diff_0.250000_Fc_0.000000_\
tau_4.000000_Uniform_bv_0.500000_sr_50.000000_Resolution_64x128/
```

**Non-Fourier Diffusion** Go to the build directory (Nova/build/), run the following command for non-Fourier diffusion test case:

```
./bin/smoke_2d -threads 8 -last_frame 300 -diff_coeff 0.25 -fc 0 -tau 4 \
-bv 0.5 -sr 50 -size 64 128 -mg_levels 2 -test_number 5
```

Set plugin to be libplugin\_Autonomous\_Navigation in Nova/build/nova.conf. Use opengl viewer to check out the cone structure:

```
./bin/opengl Implicit_Smoke_2d_NF_case_5_diff_0.250000_Fc_0.000000_\
tau_4.000000_Uniform_bv_0.500000_sr_50.000000_Resolution_64x128/
```

**Without Diffusion** Go to the build directory (Nova/build/), run the following command for smoke simulation without any diffusion:

```
./bin/smoke_3d -threads 8 -last_frame 500 -test_number 3 -bv 1 \
-sr 50 -size 256 512 256 -mg_levels 4 -nd
```

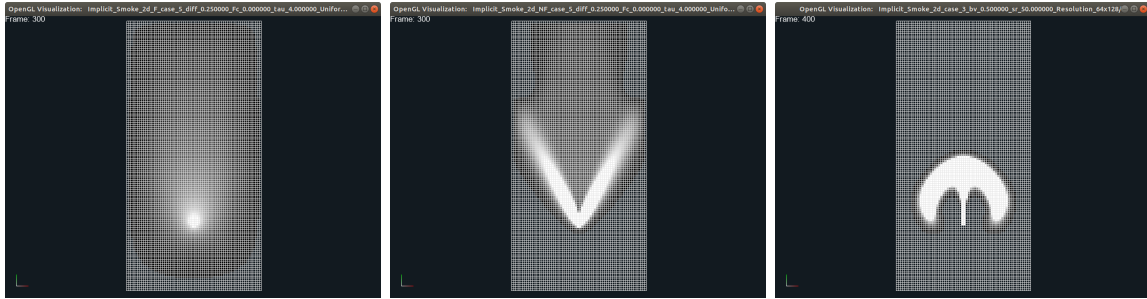


Figure 1: From left to right: Fourier diffusion, non-Fourier diffusion and without diffusion

## 2.4.2 Snowflake

**Fourier Diffusion** Go to the build directory (Nova/build/), run commands for different test cases as below:

```
./bin/snow_flake_2d -threads 8 -last_frame 200 -framerate 240 \
-omega 6 -taus 3e-4 -gamma 10 -K 2 -delta 1e-2 -eps_xy 1e-2 \
-ma 0.9 -SR 0 -rf 5e-2 -dt 1e-4 -size 256 256 -cw 3e-2 -ficks -ed
```

Set plugin to be libplugin\_Autonomous\_Navigation in Nova/build/nova.conf. Go to the build directory(Nova/build/), and run the following command to view the result:

```
./bin/opengl Snow_Flake_F_2d_case1_6branches_Resolution_256x256/
```

**Non-Fourier Diffusion** Go to the build directory (Nova/build/), run commands for different test cases as below:

```
./bin/snow_flake_2d -threads 8 -last_frame 120 -framerate 480 \
-omega 8 -taus 1e-5 -tau1 1e-2 -tau2 1e-4 -fc1 0 -fc2 0 \
-gamma 10 -K 5 -delta 1e-2 -eps_xy 1e-2 -ma 0.9 -rf 5e-2 \
-dt 1e-6 -size 256 256 -ed
```

Set plugin to be libplugin\_Autonomous.Navigation in Nova/build/nova.conf. Go to the build directory(Nova/build/), and run the following command to view the result:

```
./bin/opengl Snow_Flake_NF_2d_case1_8branches_Resolution_256x256/
```

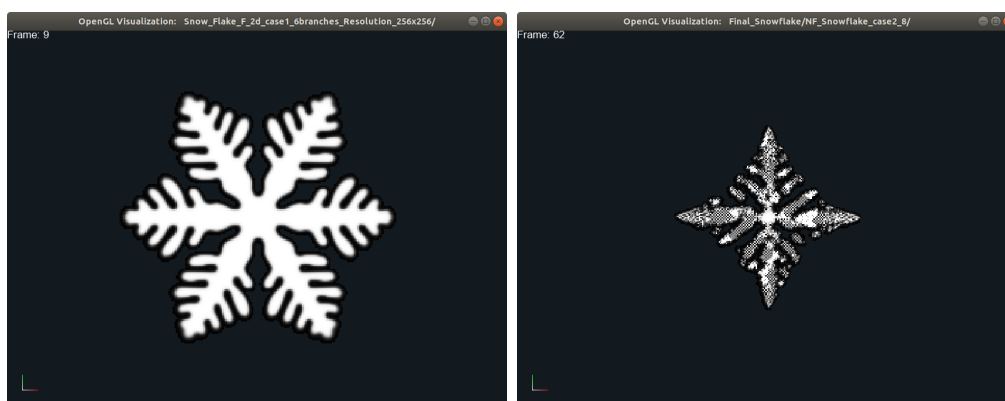


Figure 2: From left to right: Fourier snowflake and non-Fourier diffusion snowflake

### 2.4.3 Coral Reef

Go to the build directory (Nova/build/), run the following command

```
./bin/dendrite_growth_3d -threads 8 -last_frame 500 \
-ficks -ed -framerate 240 -taus 3e-4 -gamma 10 -K 4 \
-delta 1e-2 -eps 1e-2 -ma 0.9 -rf 5e-2 -dt 1e-4 \
-size 120 200 120 -cw 3e-2
```

Set plugin to be libplugin\_Autonomous.Navigation in Nova/build/nova.conf. Go to the build directory(Nova/build/), and run the following command to view the result:

```
./bin/opengl Dendrite_Growth_F_3d_case1_Resolution_120x200x120/
```

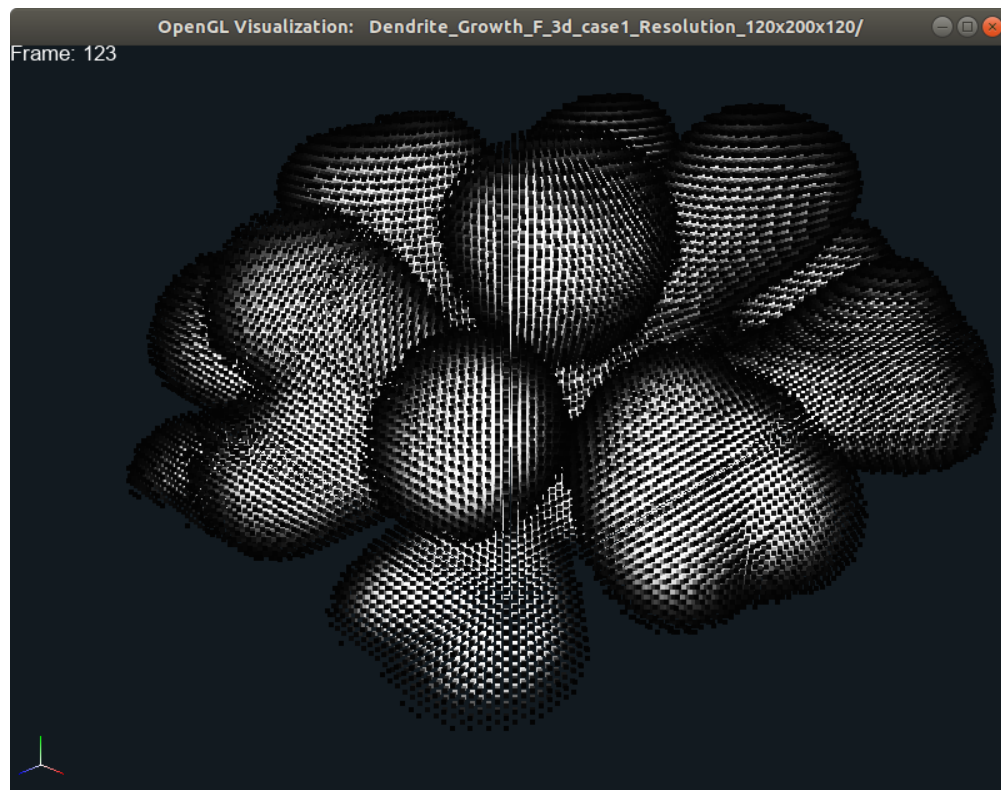


Figure 3: Coral reef