A Unified Second-Order Accurate in Time MPM Formulation for Simulating Viscoelastic Liquids with Phase Change

ANONYMOUS AUTHOR(S)

January 27, 2021

1 Description

We assume that the viscous forces in any liquid are simultaneously local and non-local, and introduce the extended POM-POM model to computer graphics to design a unified constitutive model for viscosity with several attractive properties:

1. It generalizes prior models spanning from the classical Newtonian viscous model to various non-Newtonian viscous models, such as Oldroyd-B and UCM.

2. It captures some of the most characteristic visual aspects of viscosity, such as shear thinning and shear thickening.

In the algorithm, we introduce the second-order accurate Generalized Single Step Single Solve (GS4) scheme to derive a novel second-order temporally accurate semi-implicit, partition, and explicit discretization for this viscosity model on staggered MACgrids and integrate with Material Point Method (MPM) for simulating various viscoelastic liquid behaviors.

In this document, we present the open-source MATLAB code for SIGGRAPH 2011 submission. The demonstrative algorithms include Euler Backward scheme, GS4 semi-implicit scheme, and GS4 explicit scheme. All the schemes are integrated with MPM and a 3D droplet impact case is utilized as our numerical example.

2 Compilation

The specific instruction of implementing these schemes is similar, for simplicity, we only focus on the GS4 schemes. The following instruction is tested on Windows 10 with MATLAB R2020a.

3 Input Parameters

All the input Parameters can be revised in the 'Main.m' file.

3.1 Algorithmic parameters

'eta': Semi-implicit and explicit schemes: Our algorithm can easily switch from implicit to explicit. Line 7: 'eta=1' provides semi-implicit GS4 scheme, while 'eta=0' provides explicit scheme.

'yita': Generalized solve: Our algorithm can recover the classical Newtonian solve in [1]. Line 7: 'yita=1' provides Newtonian model, while 'yita=0' provides Non-Newtonian model.

'rmin' and 'rs': Generalized time stepping schemes: Our GS4 integration recovers all prior second order accurate time integration schemes to date [2]. One can change 'rmin' (ρ_∞) and 'rs' (ρ^s_∞) in $0 \le \rho^s_\infty \le \rho_\infty \le 1$ to obtain different second-order schemes, for example:

No.	Parameters	Algorithms
1	$\rho_\infty = \rho^s_\infty = 1$	Crank-Nicolson method
2	$\rho_\infty = \rho^s_\infty = 0$	Gear's method/BDF2
3	$\rho_\infty = \rho^s_\infty$	Existing algorithms without selective control feature
4	$\rho_\infty \neq \rho^s_\infty$	New algorithms with selective control feature

Table 1: The new and existing algorithms contained in the GS4 framework

3.2 Spatial Resolutions

1. To adjust the resolution of grid discretization, one can change
   (a) Lines 15-17: computational regime
   (b) Lines 18: nodes number
2. To adjust the resolution of particle discretization, one can change
   (a) Line 53: particle numbers in each cell
   (b) Line 54: initial distribution of particles

3.3 Material Parameters

One can change materials parameters for different viscosity on Lines 8-9.

4 Demos

4.1 Newtonian Droplet Impact

Set 'yita=1' and the following material parameters

$$R_e = 0.01,\ \beta = 0.5, \alpha = 0.5, \gamma = 0.9; Q_0 = 20, W_e = 1, \Delta t = 0.0001, \Delta x = 0.1, \eta = 1$$

where R_e and W_e represent the Reynolds number and Wessienberg number, Δt nad Δx represent the temporal and spatial interval for discretization, respectively.

4.2 Non-Newtonian Droplet Impact

Case I Set 'yita=0' and the following material parameters

$$R_e = 0.01,\ \beta = 0.5, \alpha = 0.5, \gamma = 0.9; Q_0 = 20, W_e = 0.01, \Delta t = 0.0001, \Delta x = 0.1, \eta = 1$$

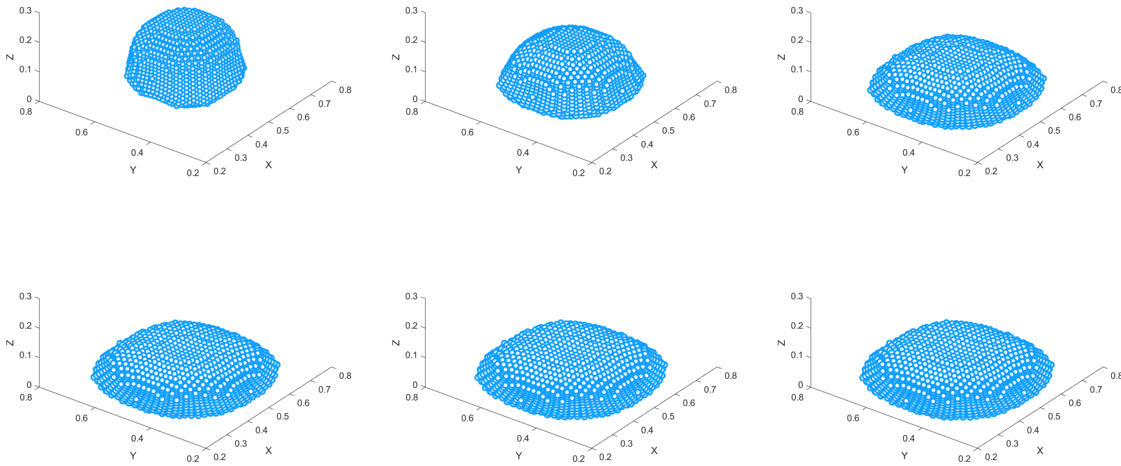

Figure 1: Newtonian Droplet at different time instant. Top: from left to right $5\Delta t$, 10 Δt, $20\Delta t$. Bottom: from left to right $30\Delta t$, 40 Δt, $50\Delta t$.

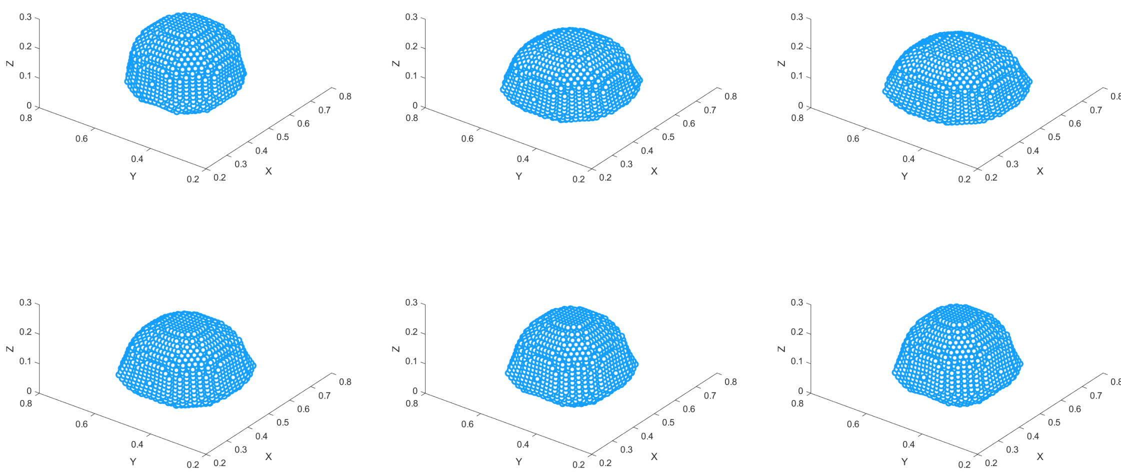

Figure 2: Non-Newtonian Droplet I with $W_e = 0.01$ at different time instant. Top: from left to right $5\Delta t$, 10 Δt, $20\Delta t$. Bottom: from left to right $30\Delta t$, 40 Δt, $50\Delta t$.

Case II Set 'yita=0' and the following material parameters

$$R_e = 0.01,\ \beta = 0.5, \alpha = 0.5, \gamma = 0.9; Q_0 = 20, W_e = 1, \Delta t = 0.0001, \Delta x = 0.1, \eta = 1$$

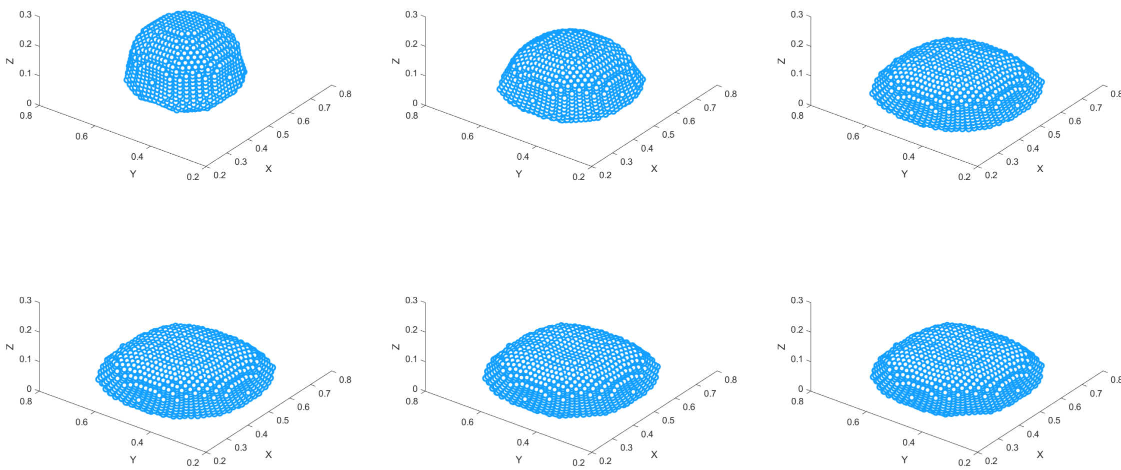

Figure 3: Non-Newtonian Droplet II with $W_e = 0.1$ at different time instants. Top: from left to right $5\Delta t$, 10 Δt, $20\Delta t$. Bottom: from left to right $30\Delta t$, 40 Δt, $50\Delta t$.

References

[1] E. Larionov, C. Batty, R. Bridson, Variational stokes: a unified pressure-viscosity solver for accurate viscous liquids, ACM Transactions on Graphics (TOG) 36 (4) (2017) 1–11.

[2] M. Shimada, S. Masuri, K. Tamma, Isochronous explicit time integration framework: illustration to thermal stress problems involving both first-and second-order transient systems, Journal of Thermal Stresses 37 (9) (2014) 1066–1079.